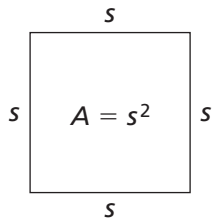
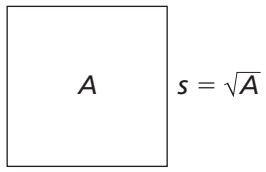
Topic 6: Cube Roots

for use after ***Looking for Pythagoras*** **Investigation 4**

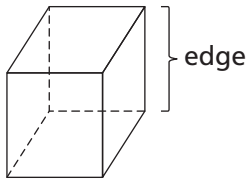
To find the area of a square, square the length of a side. When you know the area of a square, you can find the length of a side by taking the square root of the area.





Problem 6.1

A **cube** is a rectangular box with six faces that are congruent squares. The sides of the square faces are called **edges**. Volume is always measured in cubic units.



A. The formula for the volume of a rectangular box is $V = l \times w \times h$, where V is the volume, l is the length, w is the width, and h is the height.

1. What is the volume of a rectangular box with length 12 inches, width 5 inches, and height 4 inches?
2. In a cube, the length, width, and height are all equal. If the length of an edge of the cube is e, write a formula for the volume of the cube.
3. What is the volume of a cubical carton with edge length 12 inches?
4. When a number is raised to the third power, we are **cubing** the number. For example, the expression 4^3 can be read as either "4 to the third power" or "4 cubed." Why do you think we use the expression "cubing a number"?
5. Find the value of 4^3.

B. Samantha is shipping two boxes of gifts to her cousins. Each box is a cube, one with edge 7 inches and the other with edge 9 inches.

1. Write the volume of each box using exponents.
2. Calculate the volume of each box.
3. Why are the numbers you calculated called "cube numbers" or "perfect cubes"?

Problem 6.2

Finding a number when you know its cube is called finding the **cube root**. This is shown with the $\sqrt[3]{}$ symbol. For example, $\sqrt[3]{8} = 2$ because $2^3 = 8$.

A. Find the values of $\sqrt[3]{27}$ and $\sqrt[3]{125}$.

B. When you know the volume of a cube, you can find the length of the edge by taking the cube root of the volume.

1. Write a formula for finding the edge of a cube.

2. Jen and Kevin built a playhouse in the shape of a cube. The volume of the playhouse is 216 cubic feet. What are the dimensions of the playhouse?

3. A vase in the shape of a cube will hold 384 cubic inches of water when filled three-quarters of the way to the top. What is the length of an edge of the vase?

Exercises

For Exercises 1–6, find the value of each cube.

1. 5^3 **2.** 8^3 **3.** 10^3

4. 13^3 **5.** 15^3 **6.** 16^3

7. Brandon got several toys for his fourth birthday.

a. One of his gifts was a set of wooden building blocks of different sizes and shapes. Some of the blocks are cubes with edges 12 centimeters long. What is the volume of each of these blocks?

b. Brandon also got a hollow plastic Water Fun cube to use in his pool. The edges of the cube are 11 inches. How much water can Brandon pour into this cube?

For Exercises 8–13, find the value of each cube root.

8. $\sqrt[3]{1}$ **9.** $\sqrt[3]{64}$ **10.** $\sqrt[3]{216}$

11. $\sqrt[3]{729}$ **12.** $\sqrt[3]{1331}$ **13.** $\sqrt[3]{8000}$

14. A cubic shipping carton has a volume of 2744 cubic inches.

a. What is the length of an edge of this carton?

b. What is the area of each face of the carton?

Topic 6: Cube Roots

At a Glance

PACING 1 day

Mathematical Goals

- Understand relationship between cubes, cube numbers, and cube roots.
- Calculate cube numbers.
- Find cube roots of perfect cubes.

Guided Instruction

A cube is a special rectangular prism because all of its edges are the same length. You can use the formula for the volume of a rectangular prism ($V = l \times w \times h$) to develop the formula for the volume of a cube. By substituting cube side length e for l, w, and h, the formula for a cube becomes $V = e \times e \times e$, or $V = e^3$. Calculating the value of e^3 is known as cubing e. The inverse of cubing is finding the cube root, so the inverse of finding e^3 is finding $\sqrt[3]{e}$.

During Problem 6.1 A, ask:

- *Suppose you are filling the rectangular box with 1-inch cubes. How many cubes do you need to cover the bottom?* (60; Since the box is 12 in. long and 5 in. wide, I need to make 5 rows of 12 cubes each.)
- *How many cubes do you need to fill the box?* (240; Since the box is 4 in. high, I need to make 4 layers of 60 cubes each.)
- *What operation did you use?* (Multiplication)

You may also want to have students solve similar problems by filling small cubic boxes with inch cubes and finding the number of cubes in one row, one layer, and in all layers.

After Problem 6.1 A, ask:

- *What does an exponent tell you?* (How many times to use the base as a factor)

For Problem 6.2 A, ask:

- *What is the inverse of cubing a number?* (Finding the cube root)
- *How can you use an inverse operation to find the cube root of a number?* (Find the number you can use as a factor 3 times to get a product of the given number.)
- *Suppose you cannot think of the number you can cube to get a cube number. How could you find that number?* (Choose a number, cube it, then revise up or down and repeat as needed.)

For Problem 6.2 B, Part 3, ask:

- *What steps are needed to solve this problem?* (Find the volume of the entire vase. Then find the cube root of the number of units in the volume.)

You will find additional work on roots in the grade 8 unit *Looking for Pythagoras*.

Vocabulary

- cube
- edges
- cubing
- cube root

Assignment Guide for Topic 6

Core 1–14

Answers to Topic 6

Problem 6.1

A. 1. 240 cubic inches

2. $V = e^3$

3. 1728 cubic inches

4. Answers will vary. Possible answer: The volume of a cube is found by raising the length of the edge to the third power or "cubing" it.

5. $4^3 = 4 \times 4 \times 4 = 64$

B. 1. 7^3 cubic inches, 9^3 cubic inches

2. 343 cubic inches, 729 cubic inches

3. Answers will vary. Possible answer: Each of these numbers is the cube of an integer.

Problem 6.2

A. $\sqrt[3]{27} = 3$; $\sqrt[3]{125} = 5$

B. 1. $e = \sqrt[3]{V}$

2. 6 feet × 6 feet × 6 feet

3. 8 inches

Exercises

1. 125

2. 512

3. 1000

4. 2197

5. 3375

6. 4096

7. a. 1728 cubic centimeters

b. 1331 cubic centimeters

8. 1

9. 4

10. 6

11. 9

12. 11

13. 20

14. a. 14 inches

b. 196 square inches